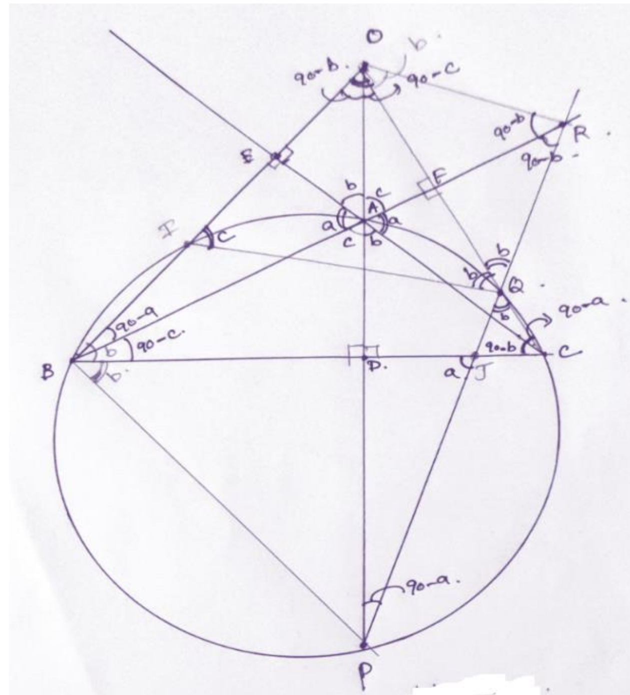
01.05.2023 – Cash Award Rider III Prize winner Mrs.Madhu Mitha's Soln

Given: ABC is a triangle.



O is the orthocentre of ΔABC.

From the given figure A is the orthocentre

for ΔOBC.

Let $\angle BOC = a, \angle OBC = b, \angle OCB = O$

$\angle ECO = 90 - a, \angle ECB = 90 - b$

$\Longrightarrow 90 - a + 90 - b = c$

$180 - (a + b) = c$

$\Longrightarrow \angle BAC = c + b$

$\angle IQO = \angle IBC = b$ ($\because$ $IQCB$ *is cyclic quardrilateral*)

$\angle OIQ = \angle QCB = C$ ($IQCB$ *is cyclic quardrilateral*)

As $\angle PAC = b = \angle PQC$ (*same segment*)

$\angle PQC = \angle OQR = b$ (*vertically opp angle*)

$\Longrightarrow OQ$ *is angle bisector of* $\angle IQR$

As $\angle PQC = b$ & $\angle QCD = c \Longrightarrow \angle QJC = a$

$\Longrightarrow \angle DJP = a \quad \Longrightarrow \angle DPJ = 90 - a$

construction: *Join* OR & IQ , BP

$\angle OBR = \angle OPR = 90 - a \Longrightarrow ORPB$ *is concyclic*

$\angle BOP = 90 - b = \angle BRP$

consider ΔOBA & ΔOBR

$\angle OBA = 90 - A , \angle BOA = 90 - b , \angle BAO = a + b$

$\angle PBC = \angle PAC = b \implies \angle PBR = 90 - c + b = \angle POR$ same segment

$\implies \angle COR = b \; as \; \angle DOC$ = 90 –c

In $\Delta OFR \; \therefore \; \angle FOR = b \implies \angle ORF = 90 - b$

Now consider $\Delta \, OBA$ & $\Delta \, OBR$

$\angle OBA = \angle OBA = 90 - a \; (common)$

$\angle BOA = \angle ORB = 90 - b$

$\angle BAO = a + b = \angle BOR = a + b$

Hence by AA similarity

$\Delta \, OBA \sim \Delta \, RBO$

$\frac{OB}{RB} = \frac{AB}{OB} = \frac{OA}{RO}$

$\implies OB^2 = AB \times RB$